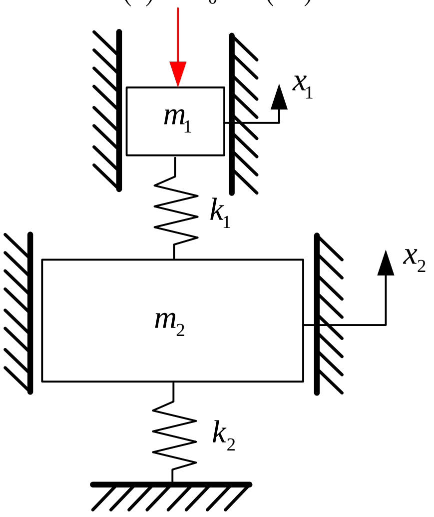
TEMAT:

Drgania wymuszone o wielu stopniach swobody (nietłumione)

Przykład 1.



$$E = \frac{1}{2} m_1 \dot{x}_1^{\,2} + \frac{1}{2} m_2 \dot{x}_2^{\,2}$$

$$U = \frac{1}{2} k_1 (x_1 - x_2)^2 + \frac{1}{2} k_2 x_2^{\,2}$$

$$\begin{cases} m_1 \ddot{x}_1 + k_1 (x_1 - x_2) = F_0 \sin(\nu t) \\ m_2 \ddot{x}_2 + k_2 x_2 - k_1 (x_1 - x_2) = 0 \end{cases}$$

$$x_1 = A_1 \sin \nu t \qquad \ddot{x}_1 = -A_1 \nu^2 \sin \nu t$$

$$x_2 = A_2 \sin \nu t \qquad \ddot{x}_2 = -A_2 \nu^2 \sin \nu t$$

$$M\ddot{x} + Kx = 0$$

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} k_1 & -k_1 \\ -k_1 & k_2 + k_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} F_0 \sin \nu t \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} (-\nu^2) + \begin{bmatrix} k_1 & -k_1 \\ -k_1 & k_2 + k_1 \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = \begin{bmatrix} F_0 \\ 0 \end{bmatrix}$$

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$$\begin{cases} (-m_1 \nu^2 + k_1) A_1 - k_1 A_2 = F_0 \\ -k_1 A_1 + (-m_2 \nu^2 + k_1 + k_2) A_2 = 0 \end{cases}$$

$$W = \begin{vmatrix} (-m_1 \nu^2 + k_1) & -k_1 \\ -k_1 & (-m_2 \nu^2 + k_1 + k_2) \end{vmatrix} = (-m_1 \nu^2 + k_1)(-m_2 \nu^2 + k_1 + k_2) - k_1^{\,2} = 0$$

$$m_1 m_2 \nu^4 - \nu^2 \left(k_1 m_1 + k_2 m_1 + k_1 m_2\right) + k_1^2 + k_1 k_2 - k_1^2 = 0$$

$$m_1 m_2 \nu^4 - \nu^2 \left(k_1 m_1 + k_2 m_1 + k_1 m_2\right) + k_1 k_2 = 0 \qquad / \cdot m_1 m_2 \qquad \nu^2 = z$$

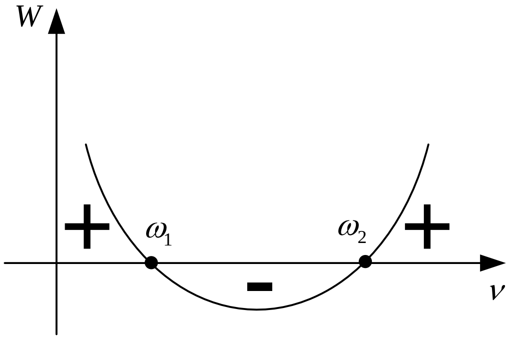
$$m_1 m_2 z^2 - z^2 \left(k_1 m_1 + k_2 m_1 + k_1 m_2\right) + k_1 k_2 = 0$$

$$\Delta = \left(\left(k_1 + k_2\right) m_1 + k_1 m_2\right)^2 - 4 k_1 k_2 m_1 m_2 = \left(k_1 + k_2\right)^2 m_1^2 + 2\left(k_1 + k_2\right) k_1 m_1 m_2 + k_1^{\,2} m_2^2 - 4 k_1 k_2 m_1 m_2$$

$$\Delta = 2 k_1^{\,2} m_1 m_2 - 2 k_1 k_2 m_1 m_2 + k_1^2 m_1^2 + 2 k_1 k_2 m_1^2 + k_2^2 m_1^2$$

$$z_1 = \frac{\left(k_1 m_1 + k_2 m_1 + k_1 m_2\right) - \sqrt{\Delta}}{2 m_1 m_2} \qquad \omega_1^2 = \frac{\left(k_1 m_1 + k_2 m_1 + k_1 m_2\right) - \sqrt{\Delta}}{2 m_1 m_2}$$

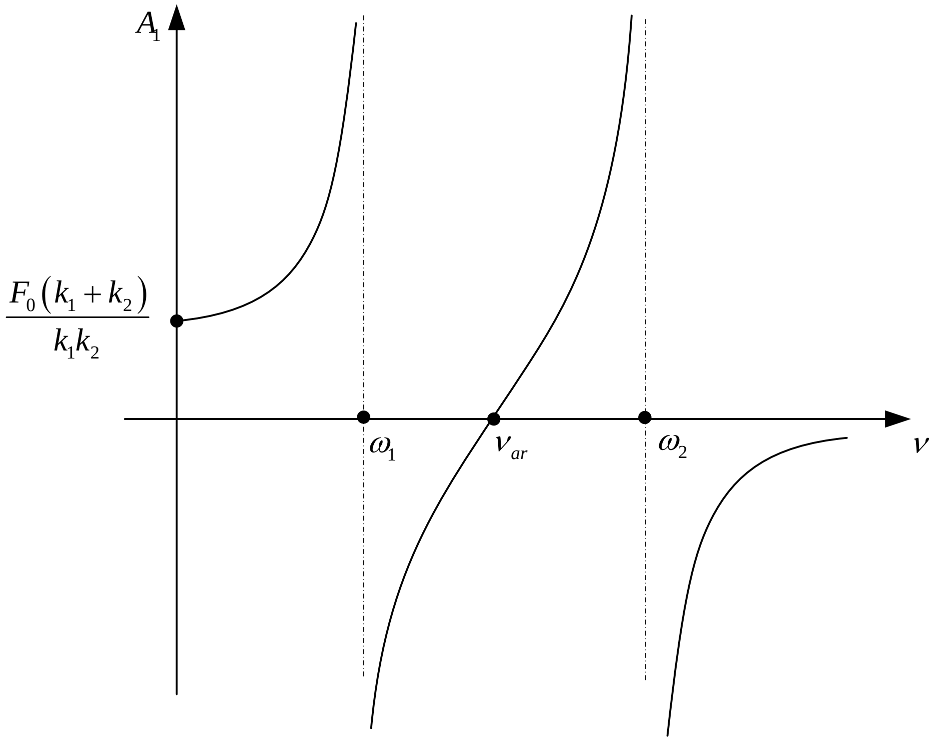
$$z_2 = \frac{\left(k_1 m_1 + k_2 m_1 + k_1 m_2\right) + \sqrt{\Delta}}{2 m_1 m_2} \qquad \omega_2^2 = \frac{\left(k_1 m_1 + k_2 m_1 + k_1 m_2\right) + \sqrt{\Delta}}{2 m_1 m_2}$$



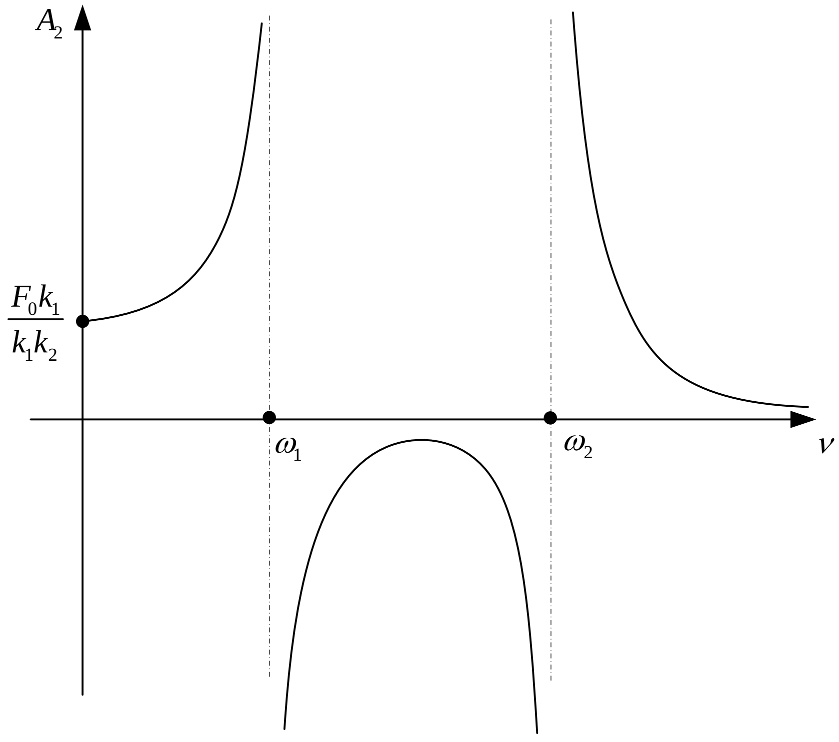
$$W_1 = \begin{vmatrix} F_0 & -k_1 \\ 0 & \left(-m_2 \nu^2 + k_1 + k_2\right) \end{vmatrix} = F_0 \left(-m_2 \nu^2 + k_1 + k_2\right)$$

$$W_2 = \begin{vmatrix} \left(-m_1 \nu^2 + k_1\right) & F_0 \\ -k_1 & 0 \end{vmatrix} = F_0 k_1$$

$A_1 = \dfrac{W_1}{W} = \dfrac{F_0\left(-m_2\nu^2 + k_1 + k_2\right)}{\left(-m_1\nu^2 + k_1\right)\left(-m_2\nu^2 + k_1 + k_2\right) - k_1^{\,2}}$	$A_2 = \dfrac{W_2}{W} = \dfrac{F_0 k_1}{\left(-m_1\nu^2 + k_1\right)\left(-m_2\nu^2 + k_1 + k_2\right) - k_1^{\,2}}$
$A_1(0) = \dfrac{F_0\left(k_1 + k_2\right)}{k_1\left(k_1 + k_2\right) - k_1^{\,2}} = \dfrac{F_0\left(k_1 + k_2\right)}{k_1 k_2}$	$A_2(0) = \dfrac{F_0 k_1}{k_1 k_2}$
$F_0\left(-m_2\nu^2 + k_1 + k_2\right) = 0$ $\nu_{ar} = \sqrt{\dfrac{k_1 + k_2}{m_2}}$ - antyrezonans	



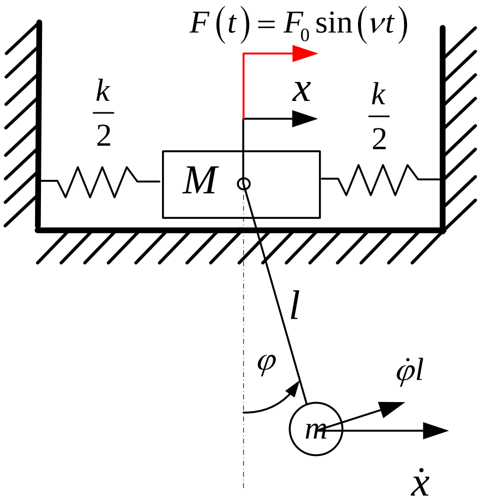
Rys. 1 Charakterystyka amplitudowo - częstotliwościowa masy pierwszej



Rys. 2 Charakterystyka amplitudowo - częstotliwościowa drugiej masy

Przykład 2.

Wyznacz częstość drgań własnych oraz charakterystyki amplitudowo częstotliwościowe dla ruchomego wahadła przedstawionego na rysunku. Przyjąc M=5m.



$$E = \frac{1}{2} M\dot{x}^2 + \frac{1}{2} m\left(\dot{\varphi} l + \dot{x}\right)^2$$

$$U = \frac{1}{2} kx^2 - mgl\cos(\varphi)$$

$$\begin{cases} M\ddot{x} + m\left(\ddot{\varphi} l + \ddot{x}\right) + kx = F_0 \sin(\nu t) \\ ml\left(\ddot{\varphi} l + \ddot{x}\right) + mgl\varphi = 0 \end{cases}$$

$$x = A_1 \sin \nu t$$
$$\varphi = A_2 \sin \nu t$$

$$\begin{bmatrix} M+m & ml \\ ml & ml^2 \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \ddot{\varphi} \end{bmatrix} + \begin{bmatrix} k & 0 \\ 0 & mgl \end{bmatrix} \begin{bmatrix} x \\ \varphi \end{bmatrix} = \begin{bmatrix} F_0 \sin \nu t \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} M+m & ml \\ ml & ml^2 \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} \left(-\nu^2\right) + \begin{bmatrix} k & 0 \\ 0 & mgl \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = \begin{bmatrix} F_0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -\nu^2 (M+m) + k & -\nu^2 ml \\ -\nu^2 ml & -\nu^2 ml^2 + mgl \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = \begin{bmatrix} F_0 \\ 0 \end{bmatrix}$$

$$W = \begin{vmatrix} -\nu^2 (M+m) + k & -\nu^2 ml \\ -\nu^2 ml & -\nu^2 ml^2 + mgl \end{vmatrix} = \left(-\nu^2 (M+m) + k\right)\left(-\nu^2 ml^2 + mgl\right) - \nu^4 m^2 l^2 = 0$$

$$\nu^4 \left(Mml^2 + m^2 l^2 - m^2 l\right) - \nu^2 \left(Mmgl + m^2 gl + mkl^2\right) + mkgl = 0$$

$$\nu^4\left(5m^2l^2\right)-\nu^2\left(6m^2gl+mkl^2\right)+mkgl=0$$

$$\nu^4-\nu^2\left(\frac{6}{5}\frac{g}{l}+\frac{k}{5m}\right)+\frac{kg}{5ml}=0$$

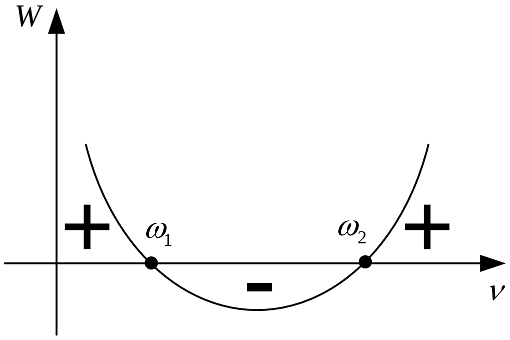
$$z^2-z\left(\frac{6}{5}\frac{g}{l}+\frac{k}{5m}\right)+\frac{kg}{5ml}=0$$

$$\Delta=\frac{36}{25}\frac{g^2}{l^2}+\frac{12}{25}\frac{kg}{ml}+\frac{k^2}{25m^2}-\frac{20kg}{25ml}=\frac{36}{25}\frac{g^2}{l^2}-\frac{8}{25}\frac{kg}{ml}+\frac{k^2}{25m^2}$$

$$\sqrt{\Delta}=\frac{1}{5m}\sqrt{36m^2\frac{g^2}{l^2}-8kgm+k^2}$$

$$\omega_1^2=\frac{\left(6gm+k\right)-\sqrt{36m^2\frac{g^2}{l^2}-8kgm+k^2}}{10m}$$

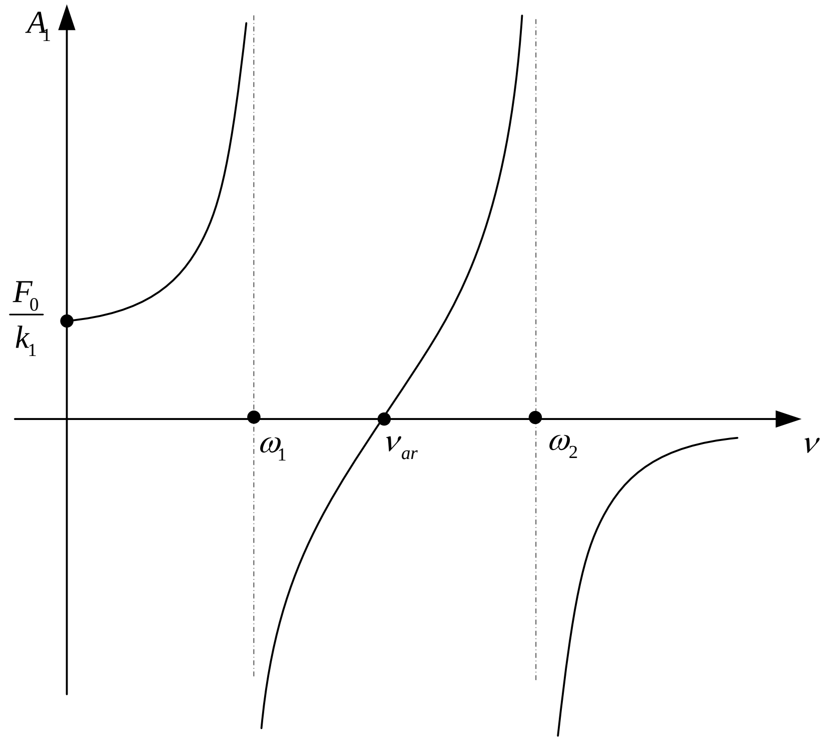
$$\omega_2^2=\frac{\left(6gm+k\right)+\sqrt{36m^2\frac{g^2}{l^2}-8kgm+k^2}}{10m}$$



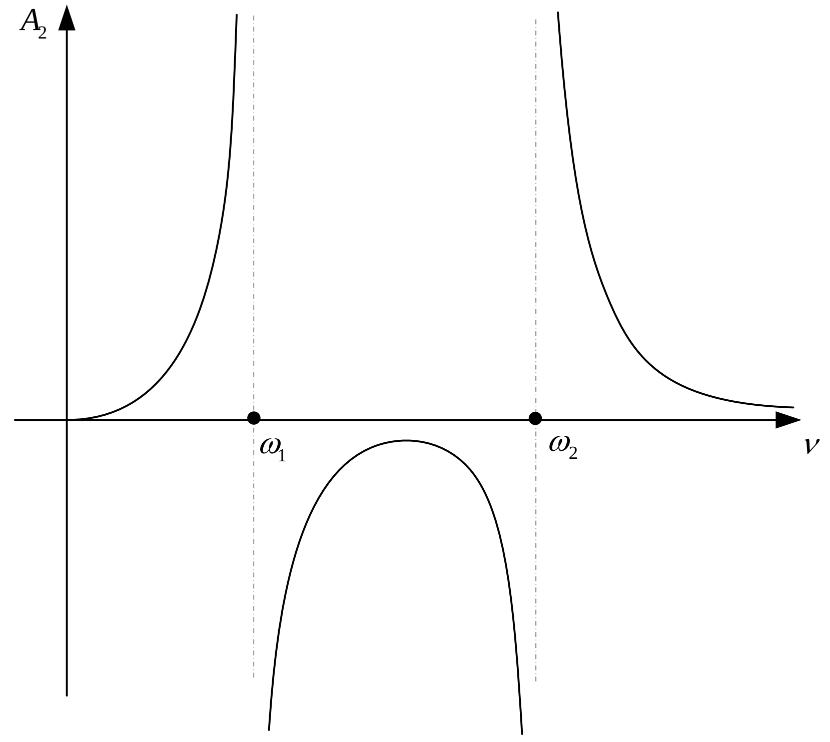
$$W_1=\begin{vmatrix}F_0 & -\nu^2ml\\ 0 & -\nu^2ml^2+mgl\end{vmatrix}=F_0\left(-\nu^2ml^2+mgl\right)$$

$$W_2=\begin{vmatrix}-\nu^2\left(M+m\right)+k & F_0\\ -\nu^2ml & 0\end{vmatrix}=F_0ml\nu^2$$

$A_1=\frac{W_1}{W}=\frac{F_0ml\left(-\nu^2l+g\right)}{\nu^4\left(5m^2l^2\right)-\nu^2\left(6m^2gl+mkl^2\right)+mkgl}$	$A_2=\frac{W_2}{W}=\frac{F_0ml\nu^2}{\nu^4\left(5m^2l^2\right)-\nu^2\left(6m^2gl+mkl^2\right)+mkgl}$
$A_1\left(0\right)=\frac{F_0mgl}{mkgl}=\frac{F_0}{k}$	$A_2\left(0\right)=0$
$F_0ml\left(-\nu^2l+g\right)=0$ $\nu_{ar}=\sqrt{\frac{g}{l}}$ - antyrezonans	



Rys. 3 Charakterystyka amplitudowo - częstotliwościowa suwaka



Rys. 4 5 Charakterystyka amplitudowo - częstotliwościowa wahadła